Portfolio Evaluation

January 9, 2014

It is Not the Return *On* My Investment ...

*“It is not the return **on** my investment that I am concerned about. It is the return **of** my investment!”*

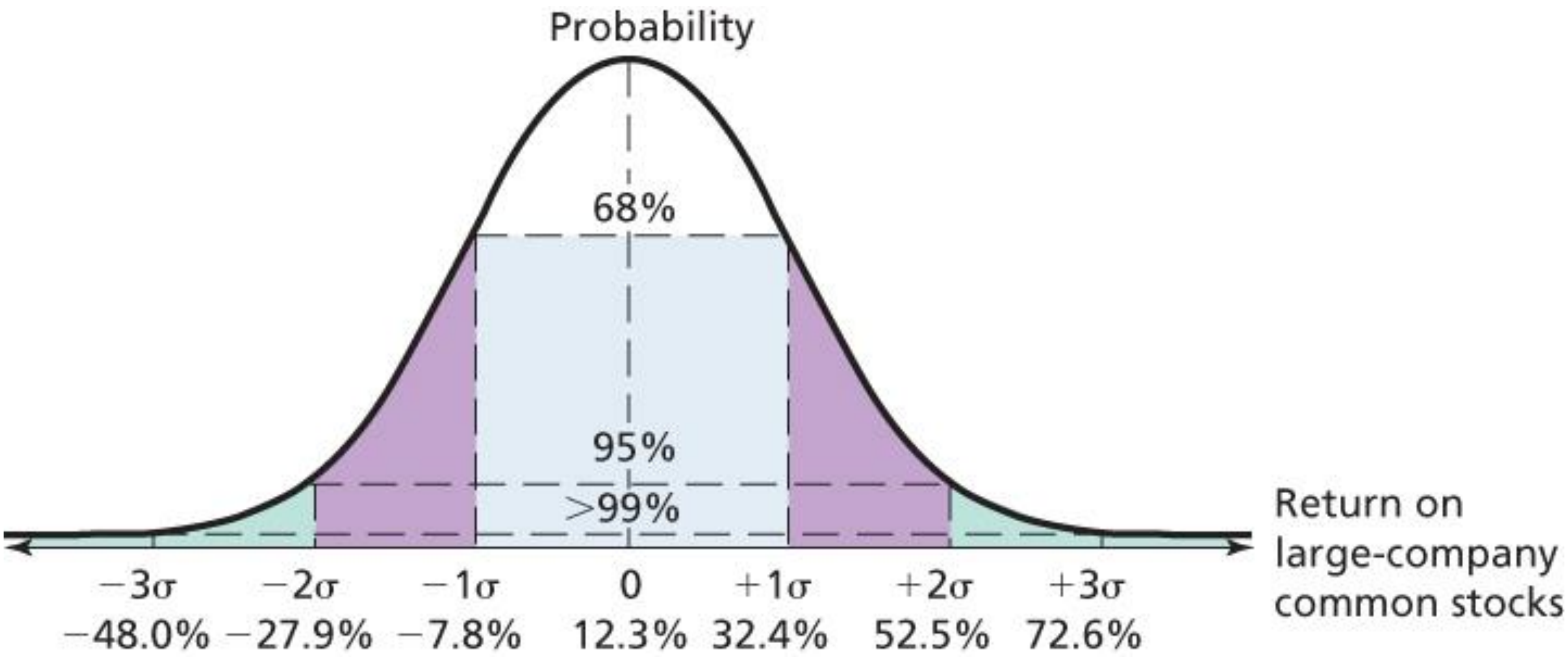
– *Will Rogers*

Objective

To get an initial evaluation of your investments' performance, you need to know:

1. How to calculate the three best-known portfolio evaluation measures.
2. The strengths and weaknesses of these three portfolio evaluation measures.
3. Understand what a Sharpe-optimal portfolio is.
4. Understand and interpret Value-at-Risk.

The Normal Distribution and Large Company Stock Returns



Performance Evaluation

- Can anyone consistently earn an “excess” return, thereby “beating” the market?
- **Performance evaluation** is a term for assessing how well a manager achieves a balance between high returns and acceptable risks.

Performance Evaluation Measures

- The ***raw return*** on a portfolio, R_P, is simply the total percentage return on a portfolio.

- The raw return is a **naive** performance evaluation measure because:
  - The raw return has no adjustment for risk.
  - The raw return is not compared to any benchmark, or standard.

- Therefore, the usefulness of the **raw return** on a portfolio is limited.

Performance Evaluation Measures

The Sharpe Ratio

- The ***Sharpe ratio*** is a reward-to-risk ratio that focuses on ***total risk.***

- It is computed as a portfolio's risk premium divided by the standard deviation for the portfolio's return.

$$\textbf{Sharpe ratio} = \frac{\mathbf{R_p - R_f}}{\boldsymbol{\sigma_p}}$$

Performance Evaluation Measures

The Treynor Ratio

- The ***Treynor ratio*** is a reward-to-risk ratio that looks at ***systematic risk only*****.**
- It is computed as a portfolio's risk premium divided by the portfolio's beta coefficient.

$$\textbf{Treynor ratio} = \frac{\mathbf{R_p - R_f}}{\boldsymbol{\beta}_\mathbf{p}}$$

Performance Evaluation Measures

Jensen's Alpha

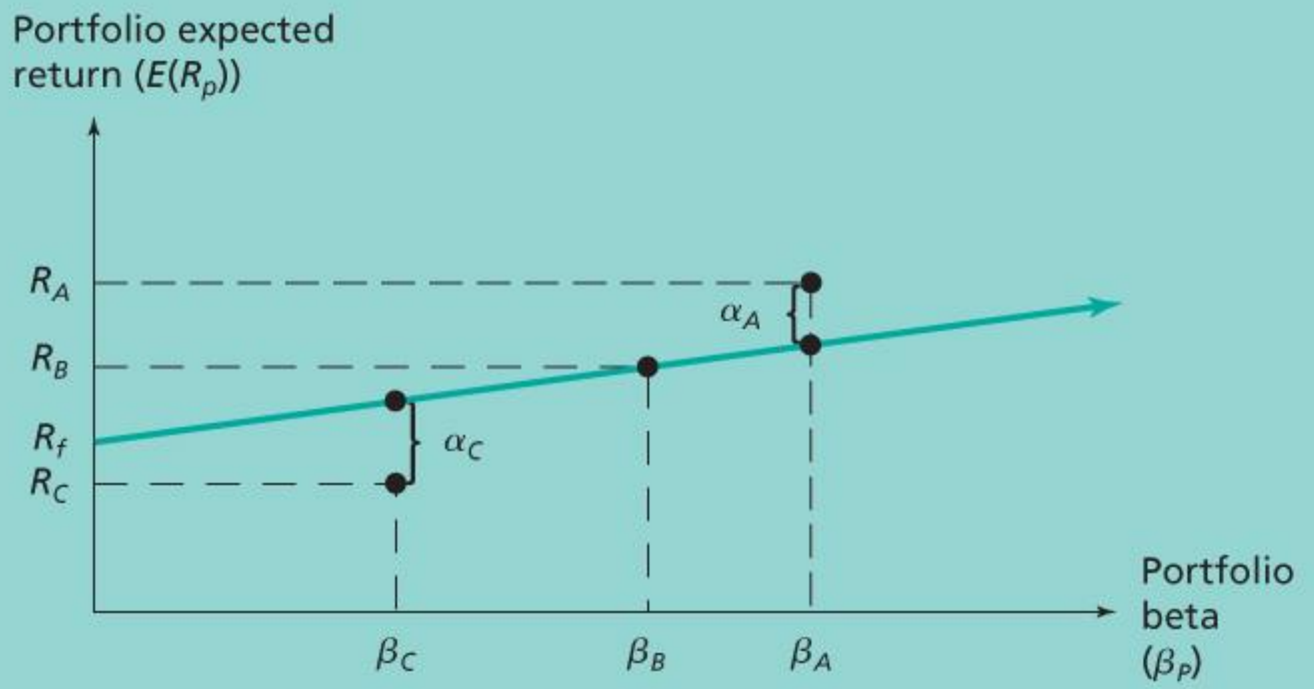
- *Jensen's alpha* is the excess return above or below the security market line. It can be interpreted as a measure of how much the portfolio "beat the market."

- It is computed as the raw portfolio return less the expected portfolio return as predicted by the CAPM.

$$\alpha_p = R_p - \left\{ R_f + \beta_p \times \left[E(R_M) - R_f \right] \right\}$$

"Extra" Return — **Actual return** — **CAPM Risk-Adjusted 'Predicted' Return**

Jensen's Alpha



Portfolio *A* plots above the Security Market Line (SML) and has a positive alpha.
Portfolio *B* has a zero alpha.
Portfolio *C* plots below the SML and has a negative alpha.

Investment Performance Data and Portfolio Performance Measurement

TABLE 13.1 Investment Performance Data

Portfolio	R_p	σ_p	β_p
A	12%	40%	.5
B	15%	30%	.75
C	20%	22%	1.4
M	15%	15%	1
F	5%	0%	0

TABLE 13.2 Portfolio Performance Measurement

Portfolio	Sharpe Ratio	Treynor Ratio	Jensen's Alpha
A	.175	.14	2%
B	.333	.133	2.5%
C	.682	.107	1%
M	.667	.10	0%

Comparing Performance Measures, I.

- Because the performance rankings can be substantially different, which performance measure should we use?

Sharpe ratio:

- Appropriate for the evaluation of an entire portfolio.

- Penalizes a portfolio for being undiversified, because in general, total risk ≈ systematic risk only for relatively well-diversified portfolios.

Comparing Performance Measures, II.

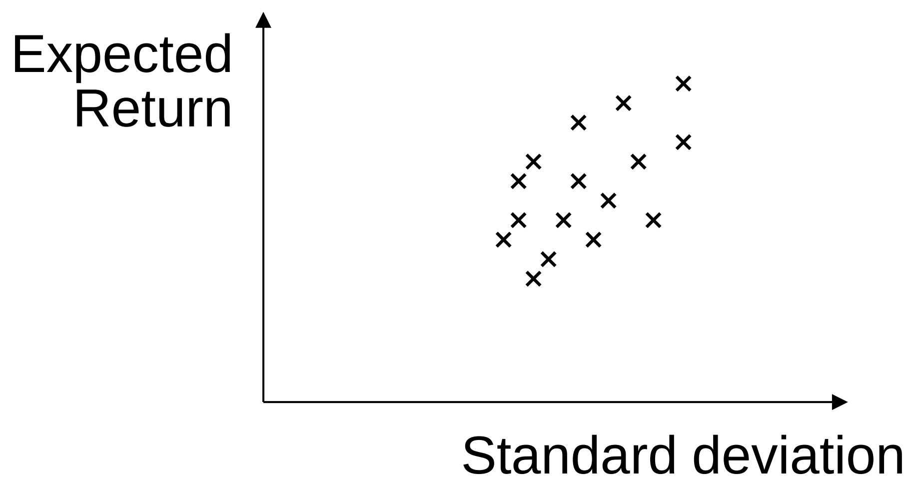
Treynor ratio and Jensen's alpha:

- Appropriate for the evaluation of securities or portfolios for possible inclusion into an existing portfolio.
- Both are similar, the only difference is that the Treynor ratio standardizes returns, including excess returns, relative to beta.
- Both require a beta estimate (and betas from different sources can differ a lot).

Sharpe-Optimal Portfolios, I.

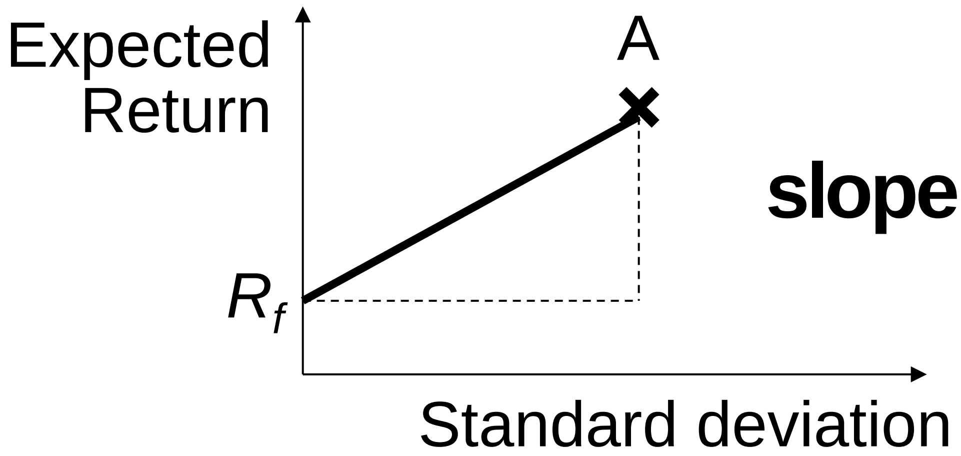
- Allocating funds to achieve the highest possible Sharpe ratio is said to be ***Sharpe-optimal***.

- To find the Sharpe-optimal portfolio, first look at the plot of the possible risk-return possibilities, i.e., the investment opportunity set.



Sharpe-Optimal Portfolios, II.

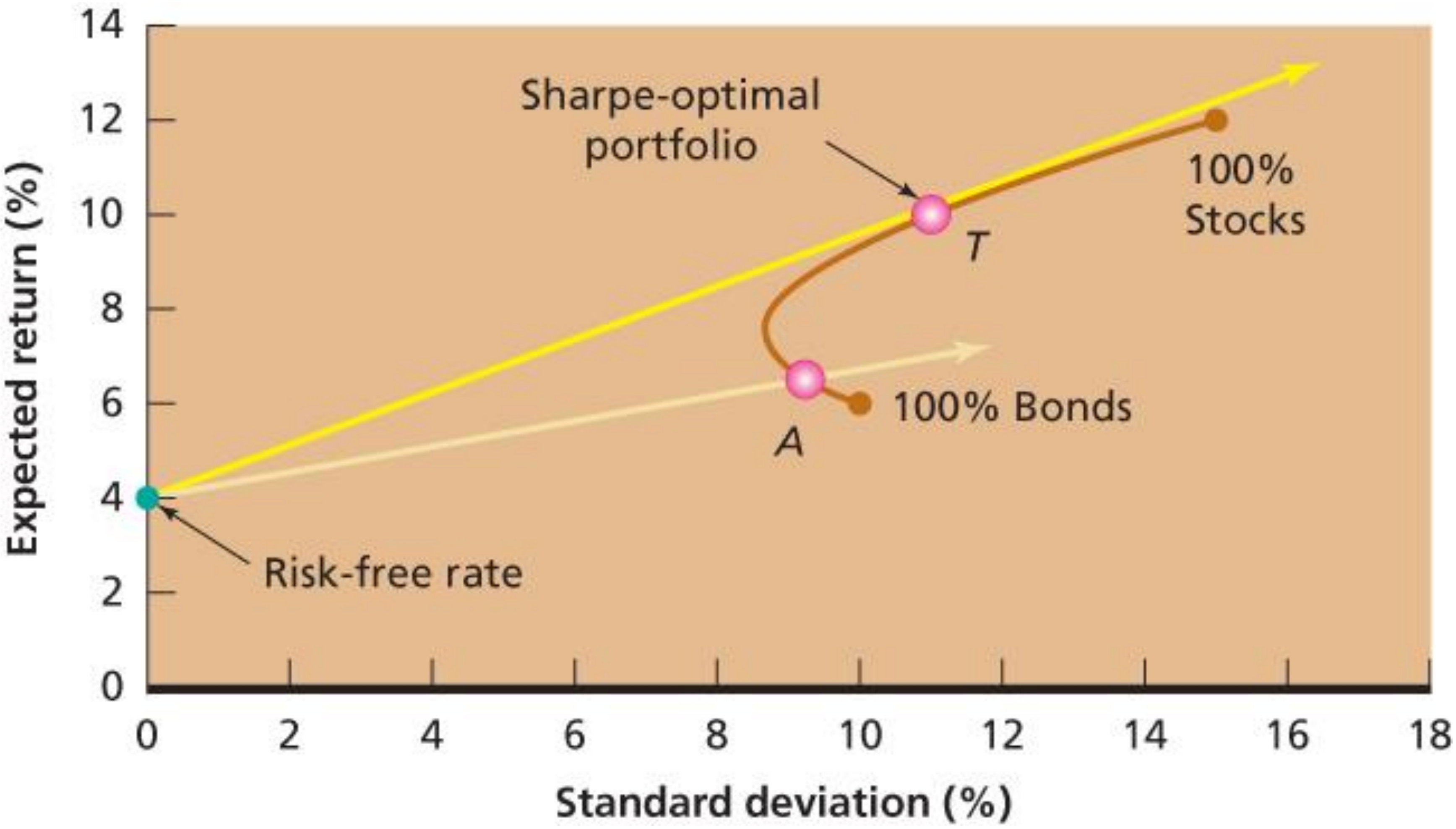
- The slope of a straight line drawn from the risk-free rate to where the portfolio plots gives the Sharpe ratio for that portfolio.



$$\textbf{slope} = \frac{E(R_A) - R_f}{\sigma_A}$$

- The portfolio with the ***steepest*** slope is the Sharpe-optimal portfolio.

Sharpe-Optimal Portfolios, III.



Example: Solving for a Sharpe-Optimal Portfolio

- For a 2-asset portfolio:

$$\text{Portfolio Return}: E(R_p) = x_s E(R_s) + x_B E(R_B)$$

$$\text{Portfolio Variance}: \sigma_P^2 = x_S^2 \sigma_S^2 + x_B{}^2 \sigma_B^2 + 2x_S x_B \sigma_S \sigma_B \text{CORR}(R_S, R_B)$$

$$\text{Sharpe Ratio} = \frac{E(R_p) - r_f}{\sigma_P} = \frac{x_S E(R_S) + x_B E(R_B) - r_f}{\sqrt{x_S^2 \sigma_S^2 + x_B^2 \sigma_B^2 + 2x_S x_B \sigma_S \sigma_B \text{CORR}(R_S, R_B)}}$$

So, we want to choose the weight in asset S that maximizes the Sharpe Ratio, using Excel.

Example: Using Excel to Solve for the Sharpe-Optimal Portfolio

Suppose we enter the data (highlighted in yellow) into a spreadsheet.

We “guess” that $X_s = 0.25$ is a “good” portfolio.

Using formulas for portfolio return and standard deviation, we compute Expected Return, Standard Deviation, and a Sharpe Ratio:

Data					
Inputs:					
ER(S):	**0.12**		**X_S:**	**0.250**	
STD(S):	**0.15**				
ER(B):	**0.06**		**ER(P):**	**0.075**	
STD(B):	**0.10**		**STD(P):**	**0.087**	
CORR(S,B):	**0.10**				
R_f:	**0.04**		**Sharpe**		
			Ratio:	**0.402**	

Example: Using Excel to Solve for the Sharpe-Optimal Portfolio, Cont.

- Now, we let Excel solve for the weight in portfolio S that maximizes the Sharpe Ratio.

- We use the Solver, found under Tools.

Solving for the Optimal Sharpe Ratio						
Given the data inputs below, we can use the						
SOLVER function to find the Maximum Sharpe Ratio:						
Data				Changer		
Inputs:				Cell:		
ER(S):	0.12		X_S:	0.700		
STD(S):	0.15					
ER(B):	0.06		ER(P):	0.102		
STD(B):	0.10		STD(P):	0.112		
CORR(A,B):	0.10					
R_f:	0.04		Sharpe			
			Ratio:	0.553	Target Cell	

Well, the “guess” of 0.25 was a tad low….

Investment Risk Management

- **Investment risk management** concerns a money manager's control over investment risks, usually with respect to potential short-run losses.

- The Value-at-Risk (VaR) approach.

Value-at-Risk (VaR)

- **Value-at-Risk (VaR)** is a technique of assessing risk by stating the probability of a loss that a portfolio may experience within a fixed time horizon.

- **If the returns on an investment follow a normal distribution**, we can state the probability that a portfolio's return will be within a certain range, if we have the mean and standard deviation of the portfolio's return.

Example: VaR Calculation

- Suppose you own an S&P 500 index fund.
- What is the probability of a return of -7% or worse in a particular year?
- That is, one year from now, what is the probability that your portfolio value is down by 7 percent (or more)?

Example: VaR Calculation, II.

- First, the historic average return on the S&P index is about 13%, with a standard deviation of about 20%.
  - A return of -7 percent is **exactly** one standard deviation below the average, or mean (i.e., 13 – 20 = -7).
  - We know the odds of being within one standard deviation of the mean are about 2/3, or 0.67.

- In this example, being **within one standard deviation** of the mean is another way of saying that:

$$\text{Prob}(13 - 20 \leq R_{S\&P500} \leq 13 + 20) \approx 0.67$$

$$\text{or Prob}(-7 \leq R_{S\&P500} \leq 33) \approx 0.67$$

Example: VaR Calculation, III.

- That is, the probability of having an S&P 500 return between -7% and 33% is 0.67.
- So, the return will be outside this range one-third of the time.
- When the return is outside this range, half the time it will be above the range, and half the time below the range.
- Therefore, we can say: Prob ($R_{S\&P500} \leq -7$) ≈ 1/6 or 0.17

Example: A Multiple Year VaR, I.

- Once again, you own an S&P 500 index fund.
- Now, you want to know the probability of a loss of 30% or more over the next two years.
- As you know, when calculating VaR, you use the mean and the standard deviation.
- To make life easy on ourselves, let’s use the one year mean (13%) and standard deviation (20%) from the previous example.

Example: A Multiple Year VaR, II.

- Calculating the two-year average return is easy, because **means are additive.** That is, the two-year average return is:

13 + 13 = 26%

- Standard deviations, however, are **not additive**.

- Fortunately, **variances are additive**, and we know that the variance is the squared standard deviation.

- The one-year variance is 20 x 20 = 400. The two-year variance is:

400 + 400 = 800.

- Therefore, the 2-year standard deviation is the square root of 800, or about 28.28%.

Example: A Multiple Year VaR, III.

- The probability of being within two standard deviations is about 0.95.
- Armed with our two-year mean and two-year standard deviation, we can make the probability statement:

$$\textbf{Prob}(26 - 2\times 28 \leq R_{S\&P500} \leq 26 + 2\times 28) \approx .95$$

or

$$\textbf{Prob}\ (-30 \leq R_{S\&P500} \leq 82) \approx .95$$

- The return will be outside this range 5 percent of the time. When the return is outside this range, half the time it will be above the range, and half the time below the range.
- **So, Prob ($R_{S\&P500} \leq -30$) ≈ 2.5%.**

Computing Other VaRs.

- In general, for a portfolio, if *T* is the number of years,

$$\mathbf{E}(\mathbf{R}_{p,T}) = \mathbf{E}(\mathbf{R}_p) \times \mathbf{T} \qquad \boldsymbol{\sigma}_{p,T} = \boldsymbol{\sigma}_p \times \sqrt{\mathbf{T}}$$

Using the procedure from before, we make make probability statements. Three very useful ones are:

$$\mathbf{Prob}\left(\mathbf{R}_{p,T} \leq \mathbf{E}(\mathbf{R}_p) \times \mathbf{T} - \mathbf{2.326} \times \boldsymbol{\sigma}_p \sqrt{\mathbf{T}}\right) = \mathbf{1\%}$$

$$\mathbf{Prob}\left(\mathbf{R}_{p,T} \leq \mathbf{E}(\mathbf{R}_p) \times \mathbf{T} - \mathbf{1.96} \times \boldsymbol{\sigma}_p \sqrt{\mathbf{T}}\right) = \mathbf{2.5\%}$$

$$\mathbf{Prob}\left(\mathbf{R}_{p,T} \leq \mathbf{E}(\mathbf{R}_p) \times \mathbf{T} - \mathbf{1.645} \times \boldsymbol{\sigma}_p \sqrt{\mathbf{T}}\right) = \mathbf{5\%}$$

Summary

- Performance Evaluation
  - Performance Evaluation Measures
    - The Sharpe Ratio
    - The Treynor Ratio
    - Jensen's Alpha
- Comparing Performance Measures
- Sharpe-Optimal Portfolios
- Investment Risk Management and Value-at-Risk (VaR)